

# Structure function evolution at next-to-leading order and beyond

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Results are presented of two studies addressing the scaling violations of deep-inelastic structure functions. Factorization-scheme independent fits to all  $ep$  and  $\mu p$  data on  $F_2$  are performed at next-to-leading order (NLO), yielding  $\alpha_s(M_Z) = 0.114 \pm 0.002_{\text{exp}} (+0.006 - 0.004)_{\text{th}}$ . In order to reduce the theoretical error dominated by the renormalization-scale dependence, the next-higher order (NNLO) needs to be included. For the flavour non-singlet sector, it is shown that available calculations provide sufficient information for this purpose at  $x > 10^{-2}$ .

## 1. Introduction

One of the important objectives of studying structure functions in deep-inelastic scattering (DIS) is a precise determination of the QCD scale parameter  $\Lambda$  (i.e., the strong coupling  $\alpha_s$ ) from their scaling violations. In this talk we briefly present results of two studies [1, 2] aiming at an improved control and a reduction of the corresponding theoretical uncertainties.

## 2. Flavour-singlet evolution in NLO [1]

The evolution of structure functions is usually studied in terms of scale-dependent parton densities and coefficient functions. In this case the predictions of perturbative QCD are affected by two unphysical scales: the renormalization scale  $\mu_r$  and the mass-factorization scale  $\mu_f$ . While the former is unavoidable, the latter can be eliminated by recasting the evolution equations in terms of observables [3]. In the flavour-singlet sector, this procedure results in

$$\frac{d}{d \ln Q^2} \begin{pmatrix} F_2 \\ F_B \end{pmatrix} = \mathcal{P}(\alpha_s(\mu_r), \frac{\mu_r^2}{Q^2}) \otimes \begin{pmatrix} F_2 \\ F_B \end{pmatrix} \quad (1)$$

with  $F_B = dF_2/d \ln Q^2$  or  $F_B = F_L$ . The kernels  $\mathcal{P}$  are combinations of splitting functions and coefficient functions which become prohibitively complicated in Bjorken- $x$  space at NLO. Thus Eqs. (1) are most conveniently treated using modern complex Mellin-moment techniques [4].

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We have performed leading-twist NLO fits to the  $F_2^p$  data of SLAC, BCDMS, NMC, H1, and ZEUS. Statistical and systematic errors have been added quadratically, the normalization uncertainties have been taken into account separately. The singlet/non-singlet decomposition has been constrained by the  $F_2^n/F_2^p$  data of NMC. The initial shapes  $F_{2,B}(x, Q_0^2)$  are expressed via standard parametrizations for parton densities at  $\mu_f = Q_0$ .

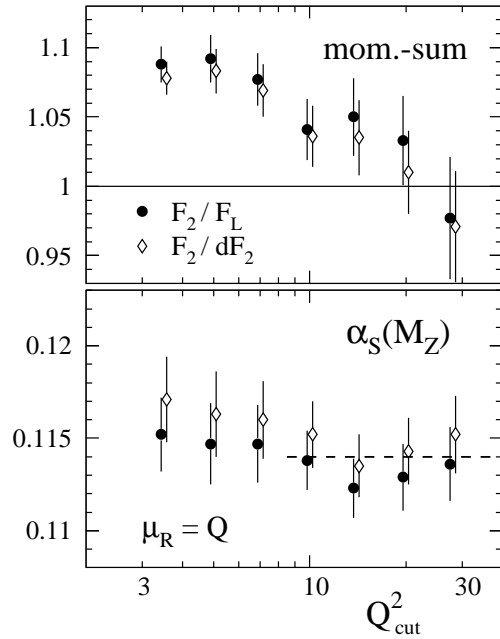


Figure 1. The dependence of the fit results for the energy-momentum sum and for  $\alpha_s(M_Z)$  on the  $Q^2$ -cut imposed in addition to  $W^2 > 10 \text{ GeV}^2$ .

In order to establish the kinematic region which can be safely used for fits of  $\alpha_s$  in the leading-twist NLO framework, the lower  $Q^2$ -cut applied to the data has been varied between 3 and 30  $\text{GeV}^2$ . When the normalized momentum sum of the partons defining the  $F_{2,B}$  initial distributions is left free, the fits with  $Q_{\text{cut}}^2 < 10 \text{ GeV}^2$  prefer values significantly different from unity, see Fig. 1. Also shown in this figure is the  $Q_{\text{cut}}^2$ -dependence of the fitted values for  $\alpha_s(M_Z)$ , now imposing the momentum sum rule. The results for  $Q_{\text{cut}}^2 \leq 7 \text{ GeV}^2$  tend to lie above the  $Q_{\text{cut}}^2 \geq 10 \text{ GeV}^2$  average of  $\alpha_s(M_Z) = 0.114$  (dashed line).

In Fig. 2 we display the renormalization scale dependence of the  $\alpha_s(M_Z)$  central values for the safe choice  $Q_{\text{cut}}^2 = 10 \text{ GeV}^2$ . The conventional, but somewhat ad hoc, prescription of estimating the theoretical error by the variation due to  $0.25 \leq \mu_r^2 / Q^2 \leq 4$  results in

$$\alpha_s(M_Z) = 0.114 \pm 0.002_{\text{exp}} \left. \begin{array}{l} +0.006 \\ -0.004 \end{array} \right|_{\text{scale}}. \quad (2)$$

Other theoretical uncertainties are considerable smaller and can be neglected at this point. The uncertainty due to possible higher-twist contributions, for instance, can be estimated at about 1% via the target-mass effects included in the fits.

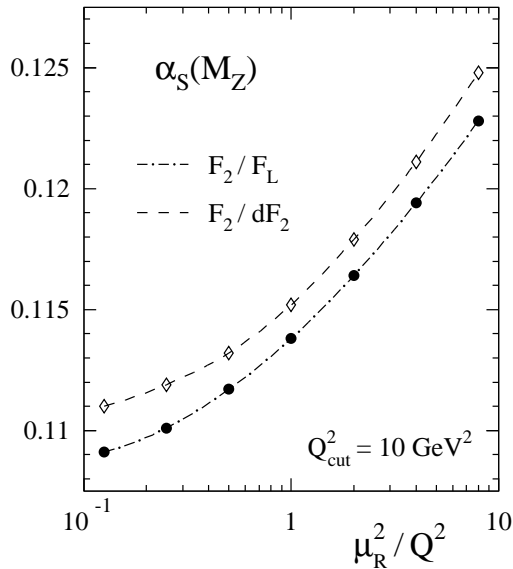


Figure 2. The dependence of the optimal values for  $\alpha_s(M_Z)$  on the renormalization scale  $\mu_r$ .

### 3. Non-singlet evolution in NNLO [ 2]

The theoretical error in Eq. (2) clearly calls for NNLO analyses. The necessary contributions to the  $\beta$ -function [ 5] and the coefficient functions [ 6] are known. However, only partial results are available for the 3-loop terms  $P^{(2)}(x)$  in the splitting-function expansion ( $a_s \equiv \alpha_s/4\pi$ )

$$P = a_s P^{(0)} + a_s^2 P^{(1)} + a_s^3 P^{(2)} + \dots \quad (3)$$

For the non-singlet part of  $F_2$  considered here ( $\text{NS}^+$ ), present information comprises the lowest five even-integer moments [ 7], the full  $N_f^2$  piece [ 8], and the most singular small- $x$  term [ 9].

We have performed a systematic study of the constraints imposed on  $P_{\text{NS}}^{(2)+}(x)$  by these results. Four approximations spanning the current uncertainty range are shown in Fig. 3, together with their convolutions with a typical input shape.

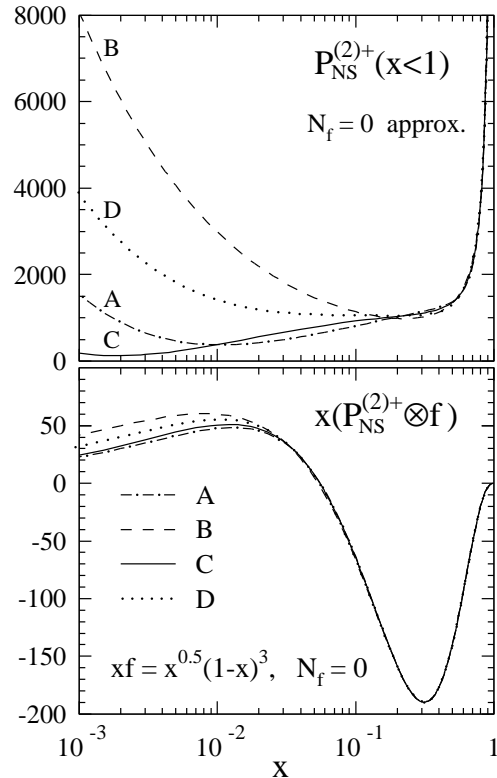


Figure 3. Representative approximate results for the flavour-number independent part of the 3-loop non-singlet<sup>+</sup>  $\overline{\text{MS}}$  splitting function.

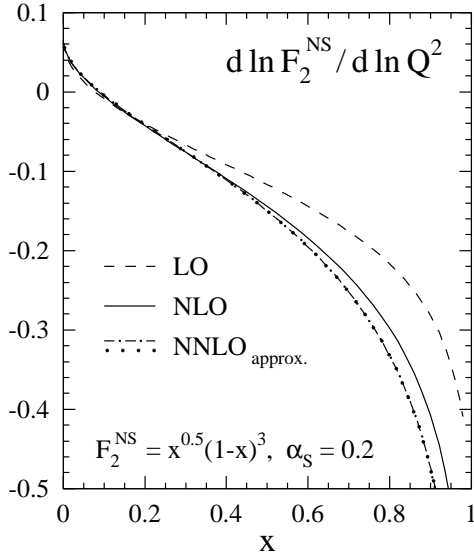


Figure 4. The first three steps in the expansion of the scaling violations of the non-singlet component of  $F_2$  for typical input parameters.

$P_{\text{NS}}^{(2)+}(x)$  is well determined at  $x \geq 0.15$ , with a total spread of about 15% at  $x \simeq 0.3$ . At (non-asymptotically) small  $x$  its behaviour is rather unconstrained despite the known leading  $x \rightarrow 0$  contribution. As the splitting functions enter scaling violations always via convolutions

$$(P \otimes f)(x) = \int_x^1 dy/y P(x/y) f(y) \quad (4)$$

with smooth initial distributions  $f(x)$ , the residual uncertainties are much reduced for observables over the full  $x$ -range. In the present case they prove to be negligible at  $x > 0.02$ .

The net effect of the NNLO correction is finally illustrated in Fig. 4, where the scale-derivative of  $F_2^{\text{NS}}$  is shown for  $\mu_r = Q$  and  $N_f = 4$ , using an  $\alpha_s$ -value typical for the fixed-target region. The inclusion of this correction into fits is expected to lead to a slightly lower central value for  $\alpha_s$  and a considerably reduced theoretical uncertainty.

#### 4. Summary and outlook

We have analyzed present  $ep/\mu p$   $F_2$ -data in a factorization-scheme independent framework [1]. We find that  $Q^2, W^2 > 10 \text{ GeV}^2$  is a safe region for

leading-twist NLO fits of  $\alpha_s$ . Our central value is close to that of the standard pre-HERA analysis in [10], but lower than the recent result of [11] using a lower  $Q^2$ -cut of  $2 \text{ GeV}^2$ . The irreducible renormalization-scale uncertainty turns out to be larger than expected from [10].

We have derived approximate  $x$ -space expressions for the 3-loop non-singlet splitting functions  $P_{\text{NS}}^{(2)}$ , including error estimates [2]. This approach is complementary to, but more flexible than, the integer-moment procedures pursued in [12, 13]. The remaining uncertainties of  $P_{\text{NS}}^{(2)}$  are small for the evolution at  $x > 10^{-2}$ , thus allowing for detailed NNLO analyses in this region. An extension to the singlet case is in preparation.

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